Theorems with names in geometry

Numerous theorems and other results in mathematics are commonly associated with names of persons. Usually such results are somehow important, and it is rewarding to acquaint oneself with their proofs. – To have a person's name associated with a result does not always indicate the person's involvement with the result.

The Theorem of Thales. The angle inscribed in a semicircle is a right angle. *Thales of Miletus*, 625–545 B.C.)

Pythagorean Theorem. In triangle ABC, the angle $\angle BCA$ is a right angle if and only if the area of the square on AB equals the sum of the areas of the squares on AC and BC. (*Pythagoras of Samos*, ca. 569–475 B.C.)

The Theorem of Archimedes or **The Broken Chord Theorem.** Let M bisect the arc $\overset{\frown}{ABC}$ of the circumcircle of triangle ABC, where $AC > BC$. If D is the orthogonal projection of M on AC, then $AD = DC + CB$. (*Archimedes of Syracuse*, 287–212 B.C.)

The Circle of Apollonios. Let AB be a line segment and k a positive constant. The the locus of points X such that $\dfrac{AX}{BX} = k$, is the circle with diameter CD, where C and D are points on the line AB satisfying $\dfrac{AC}{CB} = k$ ja $\dfrac{AD}{BD} = k$. (*Apollonios of Perga*, ca. 262–190 B.C.)

Heron's Formula. If the sides of a triangle are a, b, c and $2s = a + b + c$, then the area of the trinagle is $T = \sqrt{s(s-a)(s-b)s-c)}$. (*Heron of Alexandria.*, n. 10–75 A.D.)

The Theorem of Menelaos. Let ABC be a troiangle and let X, Y and Z be points on the lines BC, CA and AB, respectively, so that either two or none of the points are on the sides of ABC. Then X, Y and Z are collinear if and only if

$$\frac{BX}{XC} \cdot \frac{CY}{YA} \cdot \frac{AZ}{ZB} = 1.$$

(*Menelaos of Alexandria*, ca. 70–130.)

Ptolemy's Theorem. A convex quadrilateral $ABCD$is an inscribed quadrilateral if and only if $AC \cdot BD = AB \cdot CD + BC \cdot AD$. (*Claudius Ptolemy*, ca. 85–165.)

The Theorem of Pappus. If A, B, C are collinear and D, E, F are collinear, then the intersection points of AE and BD, AF and CD, and BF and CD are collinear. (*Pappus of Alexandria* (ca. 290 – ca. 350.)

Brahmagupta's Formula. If the sides of an inscribed quadrilateral are a, b, c, d, and $2s = a + b + c + d$, then the area of the quadrilateral is $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$. (*Brahmagupta*, 598–670.)

Desargues' Theorem. Let ABC and $A'B'C'$ be triangles. Then the lines AA', BB' and CC' are concurrent if and only if the points of intersection of the lines AB and $A'B'$, BC and $B'C'$, and CA and $C'A'$ are collinear. (*Girard Desargues*, 1591–1661.)

Fermat Point. If ARB, BPC and CQA are equilateral triangles on the sides of a triangle ABC, projecting outwards, then AP, BQ and CR are concurrent. (*Pierre de Fermat*, 1601–65.)

Pascal's Theorem. The points of intersection of the lines containing the opposite sides of an inscribed hexagon are collinear. (*Blaise Pascal*, 1623–62.)

Varignon's Theorem. The midpoints of the sides of a quadrilateral (or any closed broken line consisting of four segments) are vertices of a parallelogram. If the quadrilateral is convex, then the area of the parallelogram is one half of the area or the quadrilateral. (*Pierre Varignon*, 1654–1722.)

Ceva's Theorem. Let X, Y and Z be points on the sides BC, CA and AB, respectively. The lines AX, BY and CZ are concurrent if and only if

$$\frac{BX}{XC} \cdot \frac{CY}{YA} \cdot \frac{AZ}{ZB} = 1.$$

(*Giovanni Ceva*, 1674–1734.)

Simson Line. If P is a point on the circumcircle of triangle ABC, then the orthogonal projections of P on the lines AB, BC and CA are concurrent. (*Robert Simson*, 1687–1768.)

Euler Line. The intersection point of the medians, the orthocenter and the cuneter of the circuncircle of a triangle are collinear. (*Leonhard Euler*, 1707–83.)

Fagnano's Theorem. The orthotriangle has the shortest perimeter among all triangles with one vertex each on sides of a given triangle. (*Giovanni Fagnano*, 1715–97.)

Stewart's Formula. Let X be a point on side BC of a triangle ABC; let $p = AX$, $m = BX$ and $n = XC$. Then $a(p^2 + mn) = b^2m + c^2n$. (*Matthew Stewart*, 1717–85.)

Theorem of Napoleon. The centers of equilateral triangles on the sides of any triangle are vertices of an equilateral triangle. (*Napoleon Bonaparte* 1769–1821.)

Gergonne Point. The segments joining the vertices of a triangle to the points, in which the incircle of a triangle touches the opposite sides of the triangle, are concurrent. (*Joseph Diaz Gergonne*, 1771–1859.)

Feuerbach Circle or **Nine Point Circle.** The midpoints of the sides, the feet of the altitudes and the midpoints of the segments joining the orthocenter to the vertices of a triangle are on a circle. This circle is tangent to the incircle and excircles of the triangle. (*Karl Feuerbach*, 1800–34.)

Nagel Point. The segments joining the vertices of a triangle to the points in which the excircles of a triangle touch the opposite sides of the triangle are concurrent. (*Christian Heinrich von Nagel*, 1803–82)

Miquel Point. Let D, E and F be points on the sides BC, CA and AB, respectively, of a triangle ABC. The circumcircles of the triangles AFE, BDF and CED are concurrent. (*Auguste Miquel*, biographical data unknown, published 1836–46.)

Brocard Point. In any triangle ABC there is exactly one point P such that $\angle PAB = \angle PBC = \angle PCA$. (*Henri Brocard*, 1845–1922.)

Morley's Theorem. The intersection points of the trisectors of the angles of a triangle intersect each other in the vertices of an equilateral triangle. (*Frank Morley*, 1860–1937.)